Stats 2 - June 2014

① a) From calc: $\Sigma x = 1904$

$\bar{x} = 380.8$ $\quad s = 4.3817\ldots$ $\quad n = 5$

Don't know σ and $n < 30$, so need t

$v = 5 - 1 = 4$

90% t value, $v = 4$. (2 tailed) $= 2.132$

$\mu = \bar{x} \pm t \times s/\sqrt{n}$

$\mu = 380.8 \pm 2.132 \times 4.387/\sqrt{5}$

$\mu = 380.8 \pm 4.182\ldots$

$\mu = (377, 385)$

b) 90% → 10% outside → $3/30 = 3$

② a) Observed

	Eng	Scot	Wales	NI	
Male	57	44	27	17	145
Female	39	43	19	4	105
	96	87	46	21	250

b) Expected

	Eng	Scot	W	NI
Male	55.68	50.46	26.68	12.18
Female	40.32	36.54	19.32	8.82

X^2 VALUES

	Eng	Scot	W	NI
Male	0.03129	0.82702	0.00383	1.90742
Female	0.04321	1.14207	0.00530	2.63405

Test Statistic

$= \Sigma X^2$

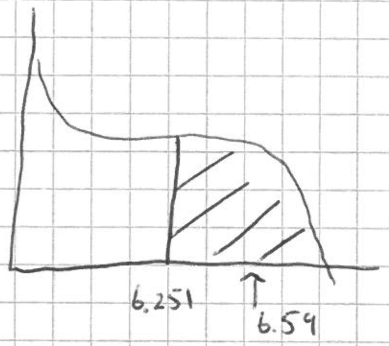
$= 6.59$

Critical Value, $v = (4-1)(2-1) = 3$, 90% level

$= 6.251$

H_0: No association between country & gender

H_1: Association between country & gender



$6.59 > 6.251$

$\therefore$ Reject H_0

There is significant evidence at 10% level of an association between country and gender of recruits.

c) More female recruits than expected recruited from Scotland (43 compared to 36.54)

③ a) $P(X \leq 4) = 0.2 + 0.1 = 0.3$

$\therefore P(\text{both } X \leq 4) = 0.3^2 = 0.09$

b) i) $0.1 + 0.2 + a + 0.3 + b = 1$

$\rightarrow a + b + 0.6 = 1$

$\rightarrow a + b = 0.4$ ①

Using $E(X)$: $3(0.1) + 4(0.2) + 5a + 6(0.3) + 7b = 5.1$

$0.3 + 0.8 + 5a + 1.8 + 7b = 5.1$

$\rightarrow 5a + 7b = 2.2$ ②

① $\times 5 \rightarrow 5a + 5b = 2$

② $\rightarrow 5a + 7b = 2.2$

② − ① $\rightarrow 2b = 0.2 \rightarrow \boxed{b = 0.1}$

① $5a + 5(0.1) = 2$

$\rightarrow 5a = 1.5 \rightarrow \boxed{a = 0.3}$

ii) $E(X^2) = 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.3 + 6^2 \times 0.3 + 7^2 \times 0.1$

$= 27.3$

$\rightarrow Var(X) = 27.3 - (5.1)^2$

$= 1.29$ QED :)

iii) $N = 2X - 5$

$E(N) = 2(E(X)) - 5$

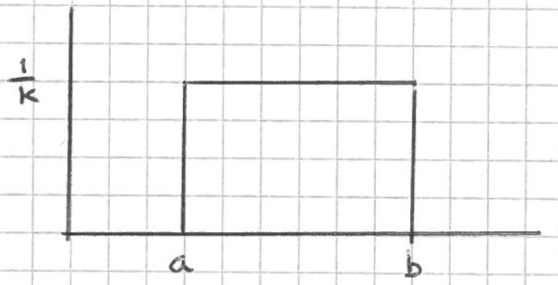
$= 2 \times 5.1 - 5 = 5.2$

$Var(N) = 2^2 \, var(X)$

$= 4 \times 1.29 = 5.16$

$\rightarrow sd(N) = \sqrt{5.16} = 2.271\ldots$

(4) a) i)



Area = 1

$\rightarrow$ $(b-a)(1/k) = 1$

$\rightarrow b - a = k$

ii) $E(X) = \frac{1}{2}(a+b)$

iii) $E(X^2) = \int_a^b x^2 f(x) \, dx$

$= \int_a^b x^2/k = \frac{1}{k}\int_a^b x^2 \, dx$

$= \frac{1}{k}\left[x^3/3\right]_a^b$

$= \frac{1}{k}\left[b^3/3 - a^3/3\right]$

we know $k = (b-a)$

$\rightarrow \left(\frac{1}{b-a}\right)\left(\frac{b^3}{3} - \frac{a^3}{3}\right) = \left(\frac{1}{b-a}\right)\left(\frac{b^3-a^3}{3}\right)$

$= \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$

$= \frac{1}{3}(b^2 + ab + a^2)$

iv) $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{1}{3}(b^2 + ab + a^2) - \left[\frac{1}{2}(a+b)\right]^2$

$= \frac{1}{3}b^2 + \frac{1}{3}ab + \frac{1}{3}a^2 - \frac{1}{4}(a+b)^2$

$= \frac{4}{12}b^2 + \frac{4}{12}ab + \frac{4}{12}a^2 - \frac{3}{12}a^2 - \frac{6}{12}ab - \frac{3}{12}b^2$

$= \frac{1}{12}b^2 - \frac{2}{12}ab + \frac{1}{12}a^2$

$= \frac{1}{12}(b^2 - 2ab + a^2)$

$= \frac{1}{12}(b - a)^2$

b) $\frac{1}{12}(b - 4)^2 = 3$

$(b - 4)^2 = 36$

$b - 4 = \pm 6$

$\Rightarrow b = 10$ or -2

b must be 10 as $b > 0$

$\Rightarrow E(x) = \frac{1}{2}(4 + 10) = 7$

(5) a) $\mu = \frac{\Sigma x}{n} = \frac{128}{40} = 3.2$ as required for λ

$s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} = \frac{126.4}{39} = 3.2410$

this is close to λ

$\therefore$ Evidence supports Peter's belief

b) $X \sim Po(3.2)$

i) $P(X > 5) = 1 - P(X \leq 5)$ (tables)

$= 1 - 0.8946 = 0.1054$

ii) $P(3 \leq X < 8)$

can be: 3, 4, 5, 6, 7

$\Rightarrow P(X \leq 7) - P(X \leq 2)$

$= 0.9832 - 0.3799 = 0.6033$

iii) $P(X \geq 1) = 1 - P(X = 0)$

$= 1 - 0.0408 = 0.9592$

P(both have 0) $= 0.0408^2$

P(1 has 0 and other has more) $= 2 \times 0.0408 \times 0.9592$

TOTAL $= 0.0408^2 + 2 \times 0.0408 \times 0.9592$

$= 0.0799$

c) $T \sim Po(5 + 3.2)$

$\rightarrow T \sim Po(8.2)$

$P(T = 9) = e^{-8.2} \times \frac{8.2^9}{9!} = 0.12686...$

$P(T = 10) = e^{-8.2} \times \frac{8.2^{10}}{10!} = 0.10403...$

TOTAL $= 0.12686... + 0.10403...$

$= 0.2308...$

6) a) $H_0: \mu = 20$

$H_1: \mu \neq 20$ (2 tailed test)

From calc: $\bar{x} = 22.625$, $s = 4.5650066...$

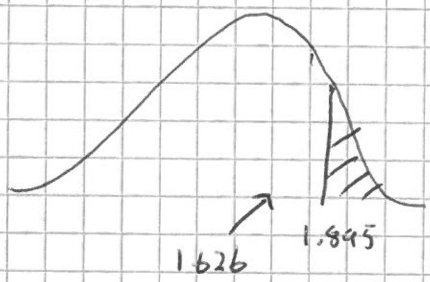
Need t distribution as $n < 30$ & don't know σ

TEST STATISTIC $= \frac{22.625 - 20}{4.565.../\sqrt{8}} = 1.626...$

CRITICAL VALUE $v = n - 1 = 8 - 1 = 7$

2 tailed, 10% sig level

$t_7(10\%) = \pm 1.895$

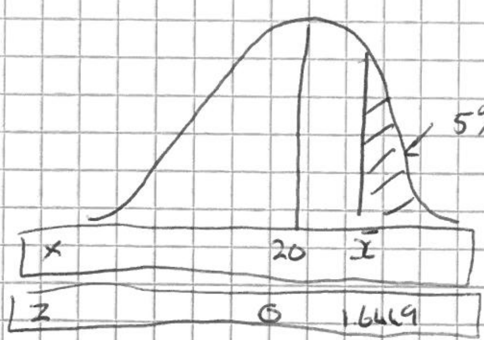


$1.626 < 1.895$

$\therefore$ Accept H_0

There is not enough evidence at the 10% significance level to support the belief that Gary does not take an average of 20 mins

b) $n > 30 \rightarrow$ use Z distribution



1 tailed as testing it is longer

Z value for 5% $= 1.6449$

Standardise: $\frac{\bar{x} - 20}{4.6/\sqrt{100}} = 1.6449$

$\rightarrow \bar{x} - 20 = 1.6449 \times 4.6/10$

$\rightarrow \bar{x} = 1.6449 \times {}^{4.6}/_{10} + 20$

$= 20.75665...$

$\therefore$ to ~~suppo~~ not support Rajul's suspicion

$\bar{x} \leq 20.75$

(7) a) $P(X<1) = \int_0^1 \frac{4}{5}x$

$= \frac{4}{5}\left[\frac{x^2}{2}\right]_0^1$

$= \frac{4}{5}\left[\frac{1}{2} - 0\right] = \frac{2}{5}$

b) i) Need $\int_1^x \frac{1}{20}(x-3)(3x-11)$

$= \frac{1}{20}\int_1^x 3x^2 - 20x + 33 \, dx$

$= \frac{1}{20}\left[x^3 - 10x^2 + 33x\right]_1^x$

$= \frac{1}{20}\left[x^3 - 10x^2 + 33x - 1 + 10 - 33\right]$

$= \frac{1}{20}\left[x^3 - 10x^2 + 33x - 24\right]$

Now need $F(1) = \frac{2}{5}$

$\therefore F(x) = \frac{1}{20}(x^3 - 10x^2 + 33x - 24) + \frac{2}{5}$

$= \frac{1}{20}(x^3 - 10x^2 + 33x - 24) + \frac{1}{20}(8)$

$= \frac{1}{20}(x^3 - 10x^2 + 33x - 24 + 8)$

$= \frac{1}{20}(x^3 - 10x^2 + 33x - 16)$ QED

ii) For median, $F(x) = 0.5$

$F(1.13) = \frac{1}{20}(1.13^3 - 10(1.13)^2 - 33(1.13) - 16)$

$= 0.49819...$

$F(1.14) = \frac{1}{20}(1.14^3 - 10(1.14)^2 - 33(1.14) - 16)$

$= 0.50527...$

$\therefore 1.13 <$ Median 1.14